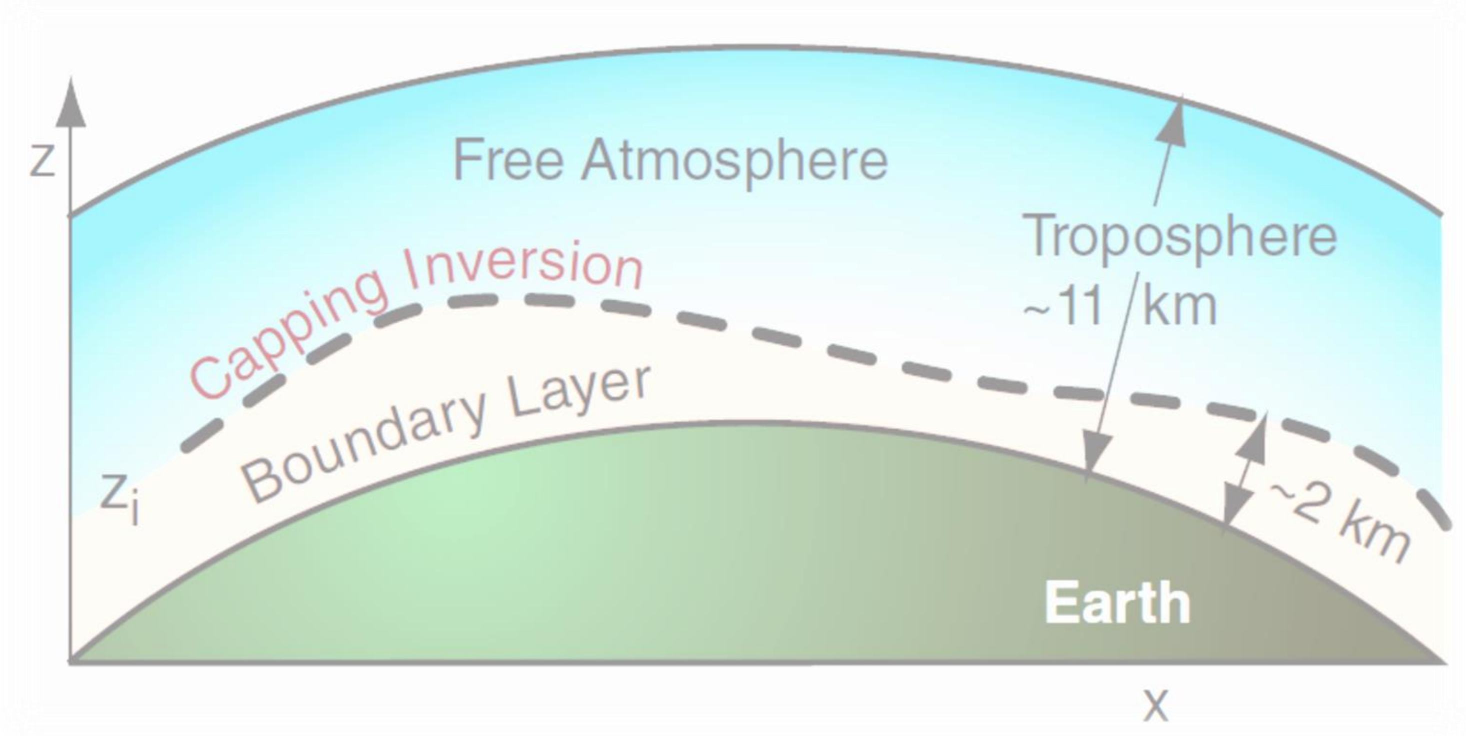
Igor Esau

Atmospheric boundary layers: Processes, modelling and challenges

University in Tromsø – Norwegian Arctic University
igor.ezau@uit.no



What is an Atmospheric Boundary Layer (ABL)?

- Source: Atmospheric boundary layer - Glossary of Meteorology (ametsoc.org)
- ABL *also called* boundary layer, planetary boundary layer.
- The bottom **[turbulent]** layer of the troposphere that is in contact with the surface of the earth.
- It is ~~**often**~~ **[always]** turbulent and is capped by a statically stable layer of air or **[potential]** temperature inversion.
- The ABL depth (i.e., the inversion height) is variable in time and space, ranging from tens of meters in strongly statically stable situations, to several kilometers in convective conditions over deserts.
- *During fair weather over land, the ABL has a marked diurnal cycle. During daytime, a mixed layer of vigorous turbulence grows in depth, capped by a statically stable entrainment zone of intermittent turbulence. Near sunset, turbulence decays, leaving a residual layer in place of the mixed layer. During nighttime, the bottom of the residual layer is transformed into a statically stable boundary layer by contact with the radiatively cooled surface. Cumulus and stratocumulus clouds can form within the top portion of a humid ABL, while fog can form at the bottom of a stable boundary layer. The bottom 10% of the ABL is called the surface layer.*
- *Stull, R. B. 1988. An Introduction to Boundary Layer Meteorology. 666 pp.*

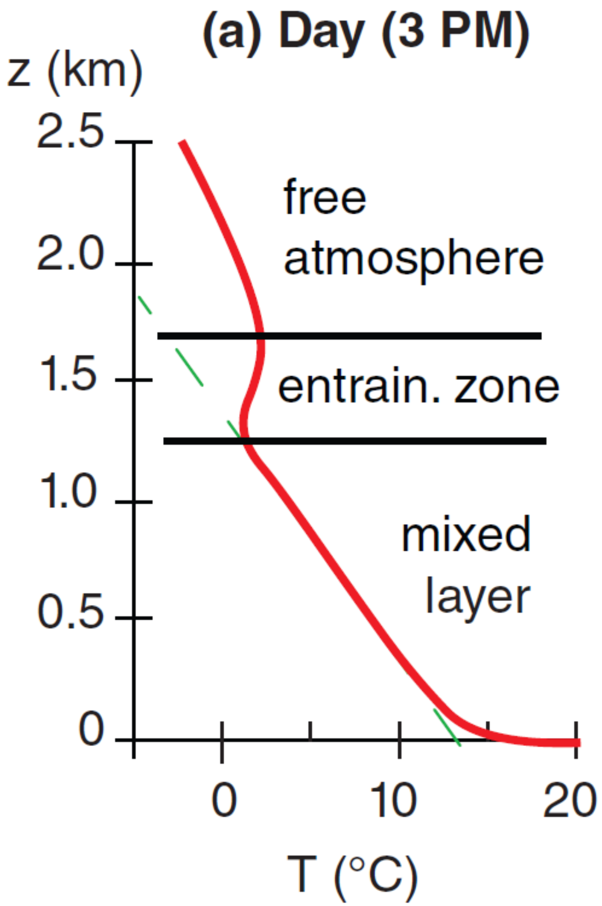
Key words:

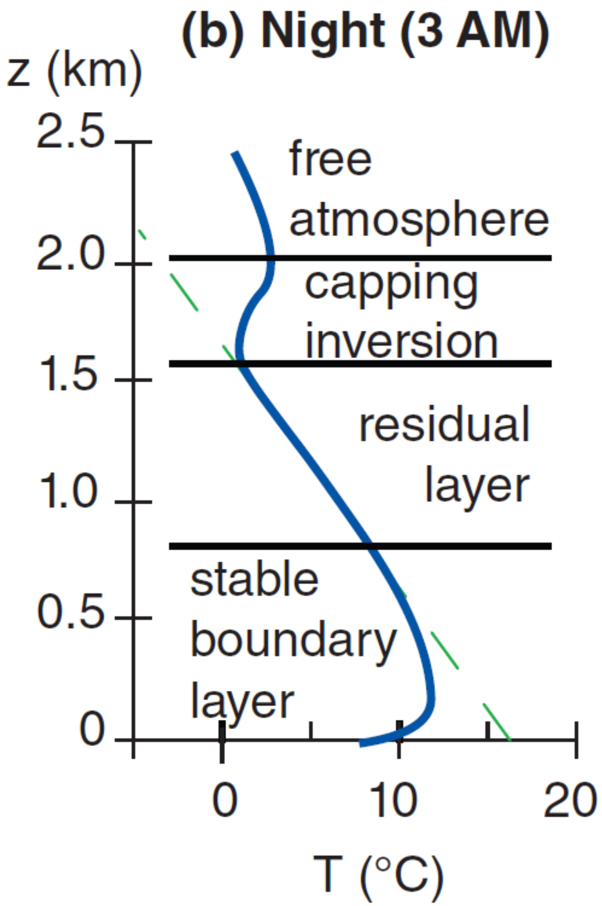
- Turbulence
- Wind and Temperature (density) profiles
- Stratification/Stability
- Surface layer or a layer of constant fluxes
- Capping potential temperature inversion

Stull, R., 2017. Practical Meteorology: An Algebra-based Survey of Atmospheric Science., University of British Columbia.

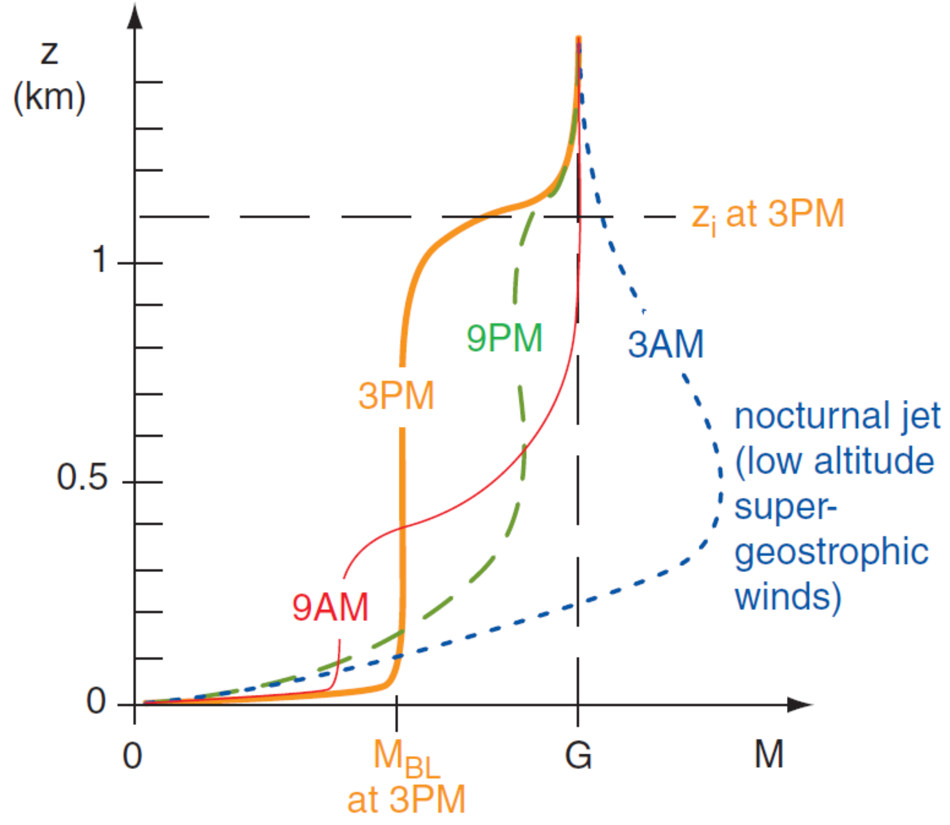
ABL structure

Absolute temperature

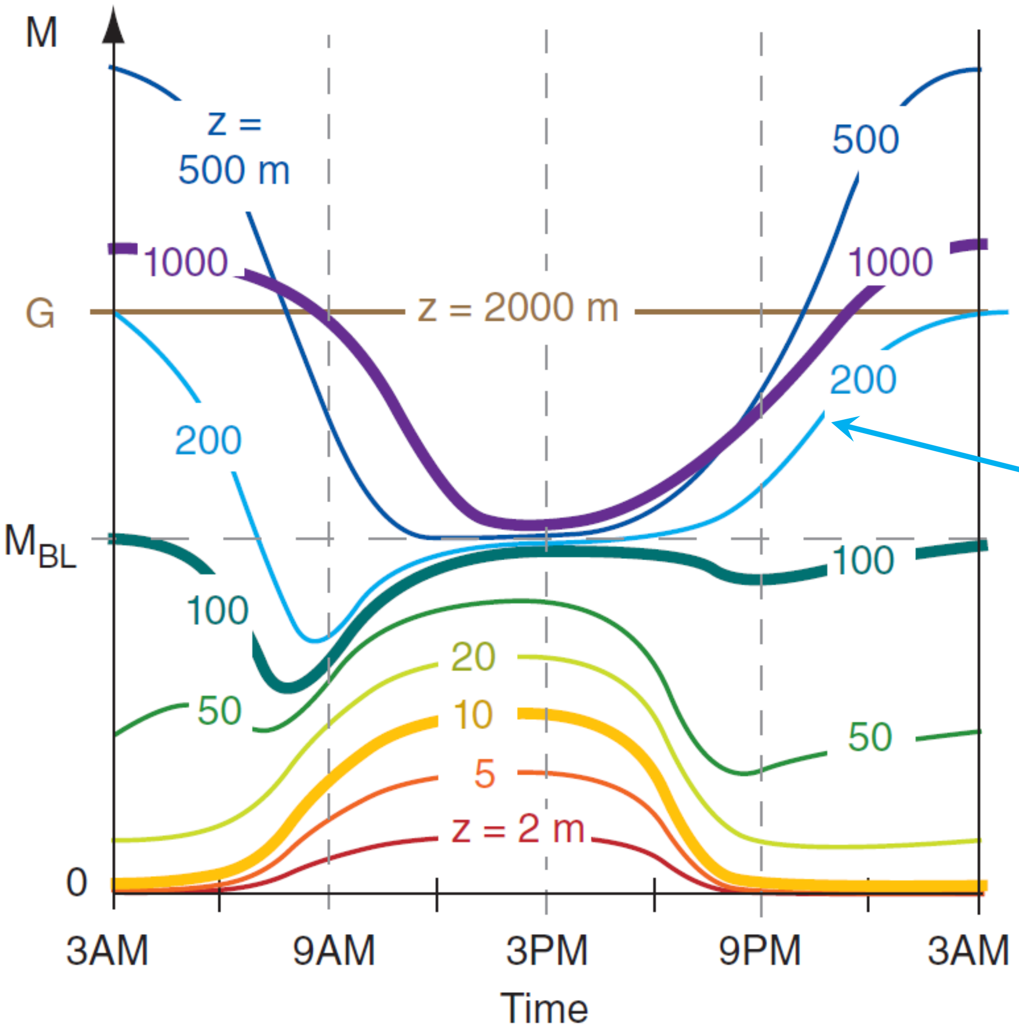




Wind speed

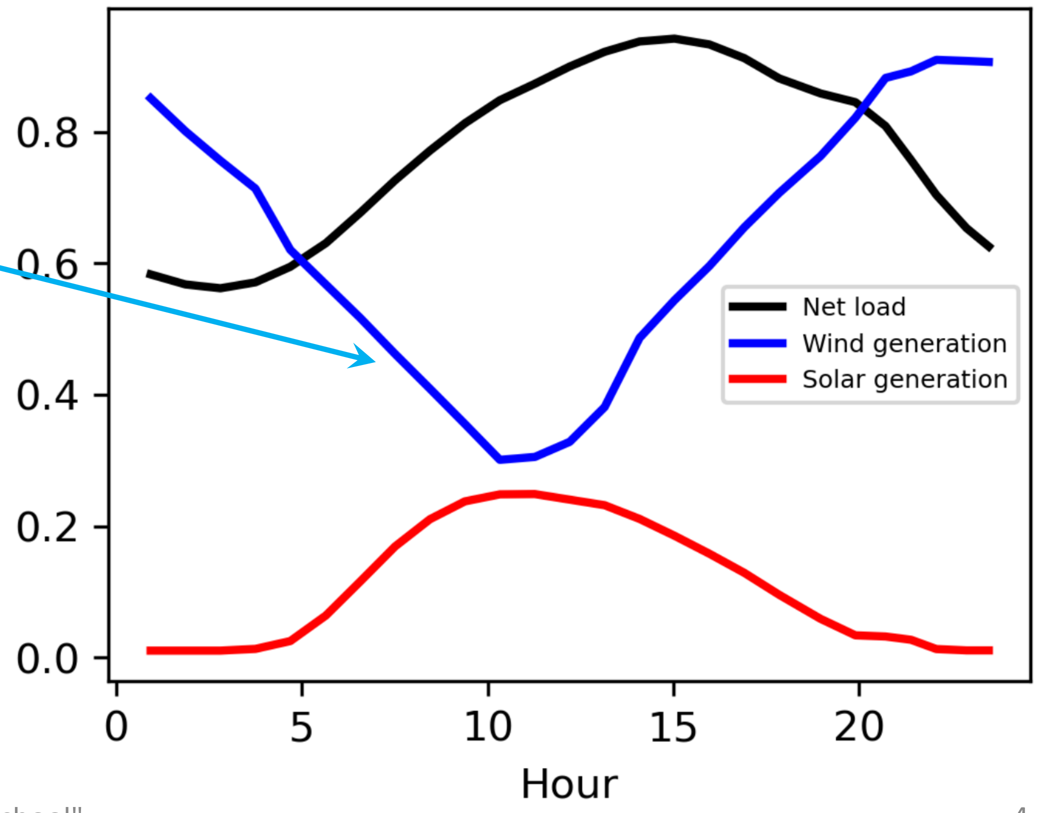


ABL structure: An example of implications



Averaged diurnal cycle (April, 2009; NREL data) of electricity net load and power generation by wind and solar installations.

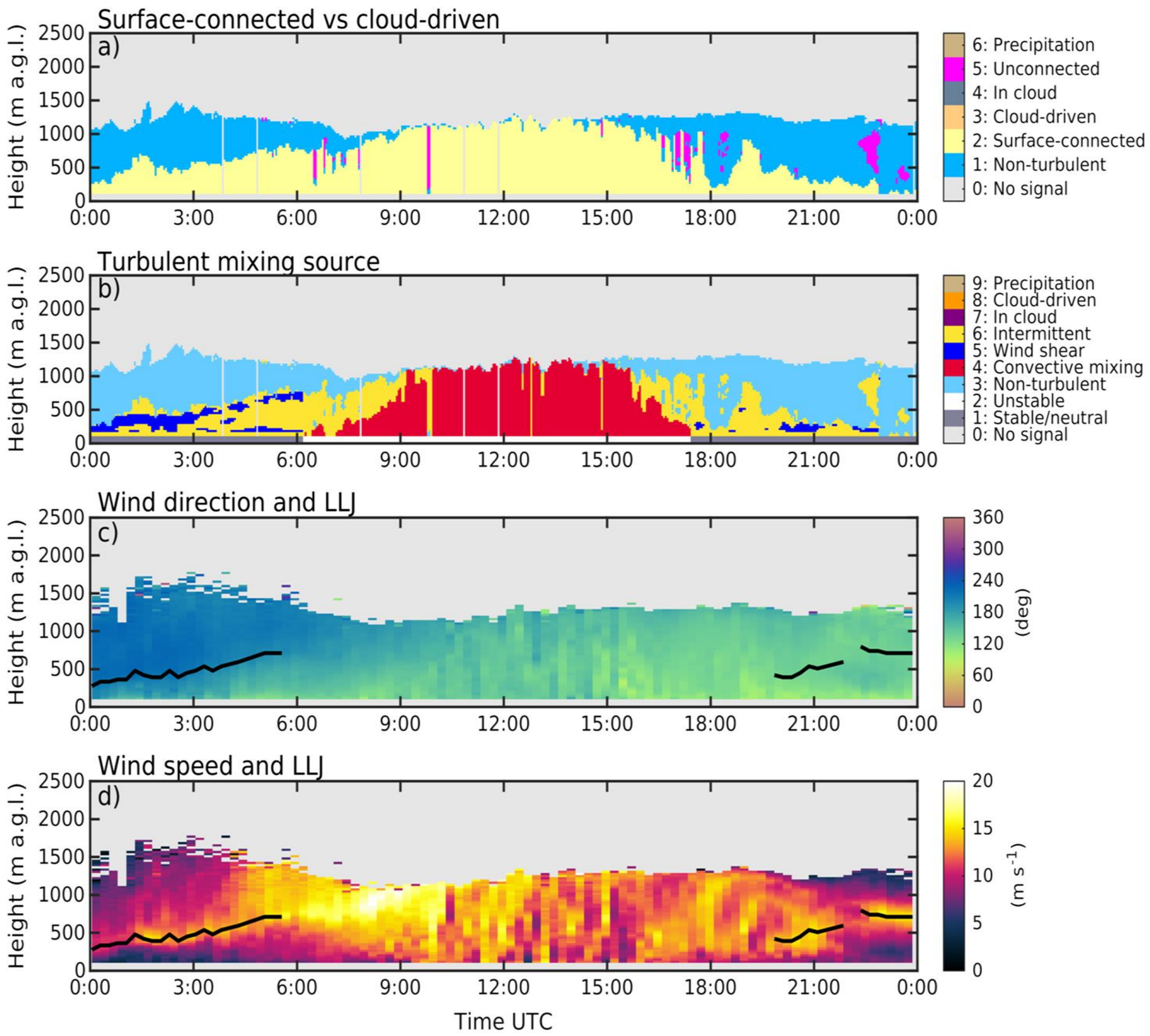
*All data are normalized with corresponding maximum values.



ABL observed with an atmospheric lidar

Manninen, A.J., Marke, T., Tuononen, M., O'Connor, E.J., 2018. Atmospheric Boundary Layer Classification With Doppler Lidar. J. Geophys. Res. Atmos. 123, 8172–8189. https://doi.org/10.1029/2017JD028169

Time-height plots of atmospheric boundary layer classification showing
(a) connection with the surface (i.e., surface driven versus cloud driven) and
(b) the turbulent mixing source, together with time-height plots of
(c) wind direction and
(d) wind speed on 9 March 2016 at Jülich, Germany.
The black lines on the two lower panels show altitude of a low-level jet (LLJ).



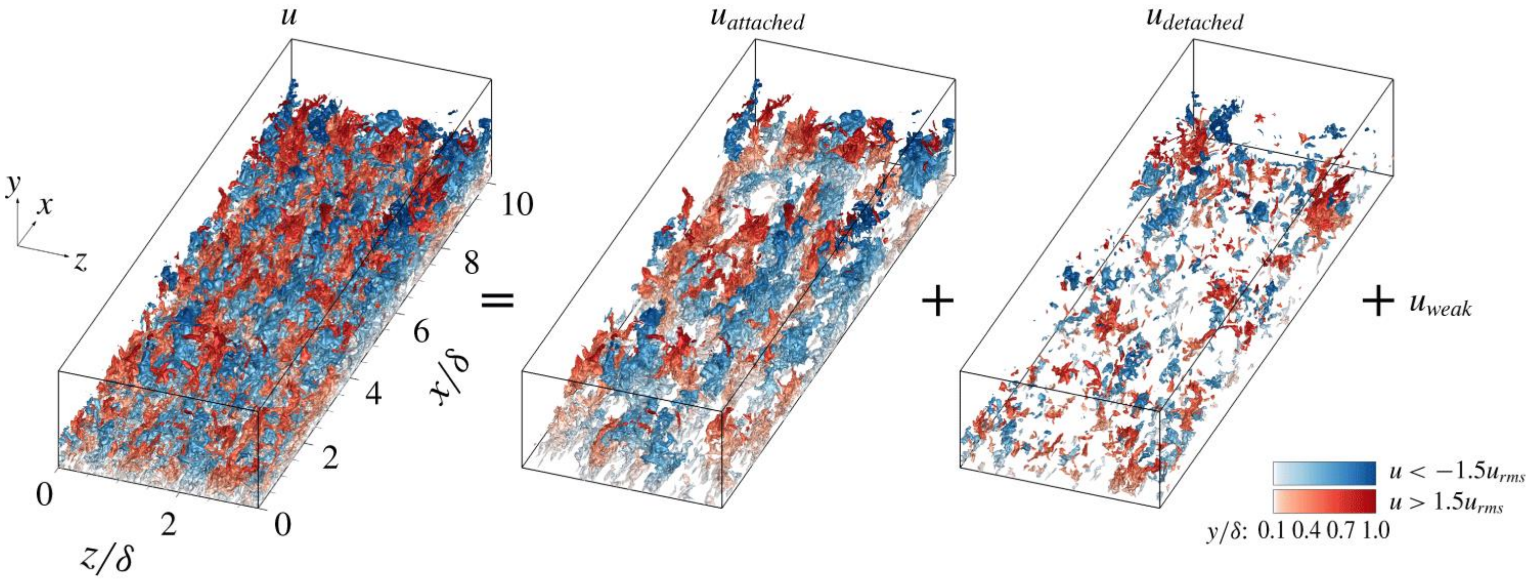
Turbulent mixing is the key dynamical process in the atmospheric boundary layer

In fluid dynamics, turbulence is a process characterized by chaotic changes in fluid motions.

Richard Feynman has described turbulence as the most important unsolved problem in classical physics.

$$Re = \frac{UL}{\nu} \gg O(10^5) \gg Re_{cr} = O(10^3)$$

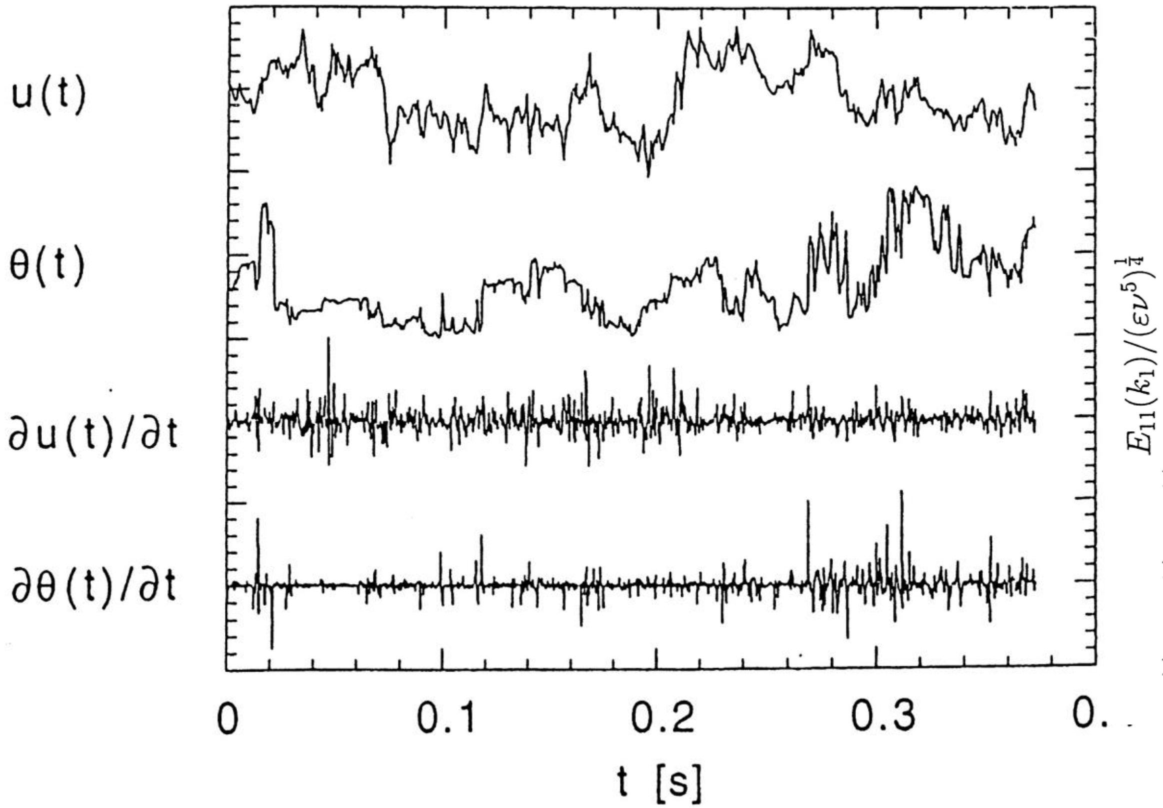
The atmosphere and the ocean are always turbulent, but there are "strong" (in ABL/OML and in some other conditions) and "weak" turbulent regimes

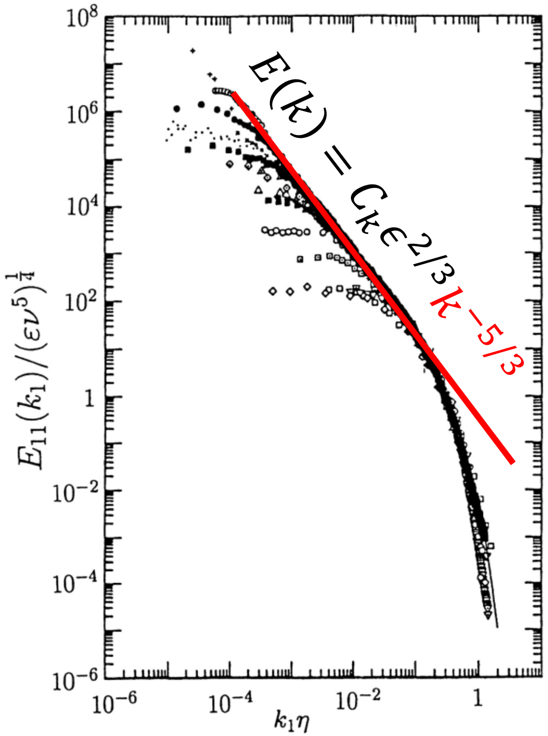


Yoon, M., Hwang, J., Yang, J., Sung, H.J., 2020. Wall-attached structures of streamwise velocity fluctuations in an adverse-pressure-gradient turbulent boundary layer. J. Fluid Mech. 885, A12. https://doi.org/10.1017/jfm.2019.950

ABL in nature

Chaotic process requires statistical approach





Fluctuations are chaotic but not completely random

- Not white noise
- Large, energic fluctuations are considerably more frequent than one may expect having a Gaussian process

Temperature fluctuations significantly more skewed as compared to velocity fluctuations

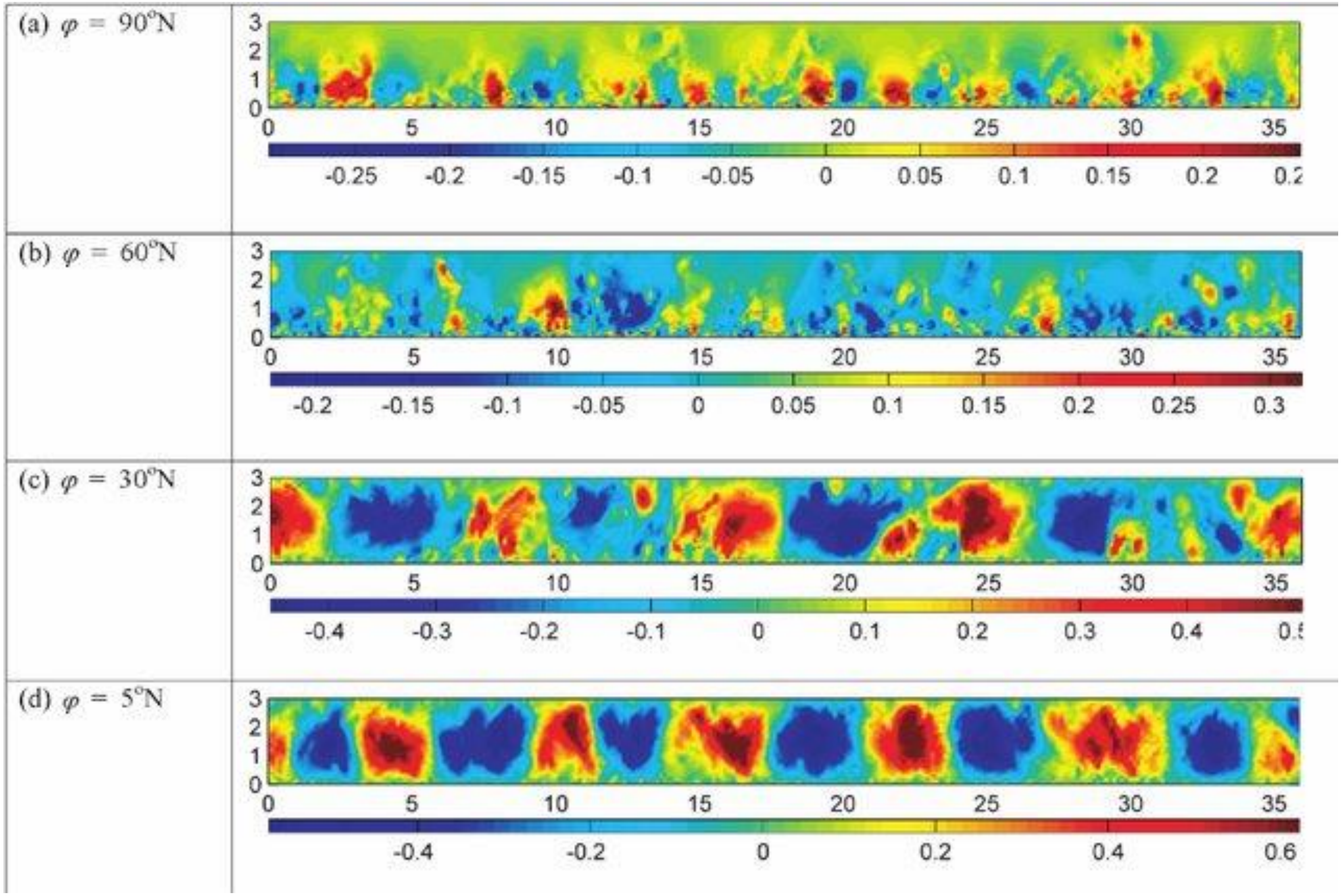
Turbulence could be divided on

- a universal (Kolmogorov) part
- A non-universal (self-organized) part

Time series of the longitudinal velocity fluctuations, $u(t)$, and their derivative, $\partial u(t)/\partial t$. Also shown is the time series of scalar fluctuations, $\theta(t)$, and their derivative, $\partial\theta(t)/\partial t$, in the same flow. Notice the higher intermittency in the scalar (bottom trace). $R_\lambda = 582$. Measurements by L. Mydlarski

Warhaft, Z., 2002. Turbulence in nature and in the laboratory. Proc. Natl. Acad. Sci. 99, 2481–2486. https://doi.org/10.1073/pnas.012580299

Self-organized turbulence in the PBL



Self-organized turbulence in the PBL

Table 2-1 Some typical parameters of open cell convection (Atkinson and Zhang, 1996; Agee and Lomax, 1978)

Cell diameter	30 km
Cell height	2 km
Cell aspect ratio (diameter/height)	15
Tsea-Tair	2-5 °C
Low level wind shear (direction)	< 7 ° km^{-1}
Low level wind shear (speed)	< 2 m s^{-1} km^{-1}
Cloudiness	~ 40 %

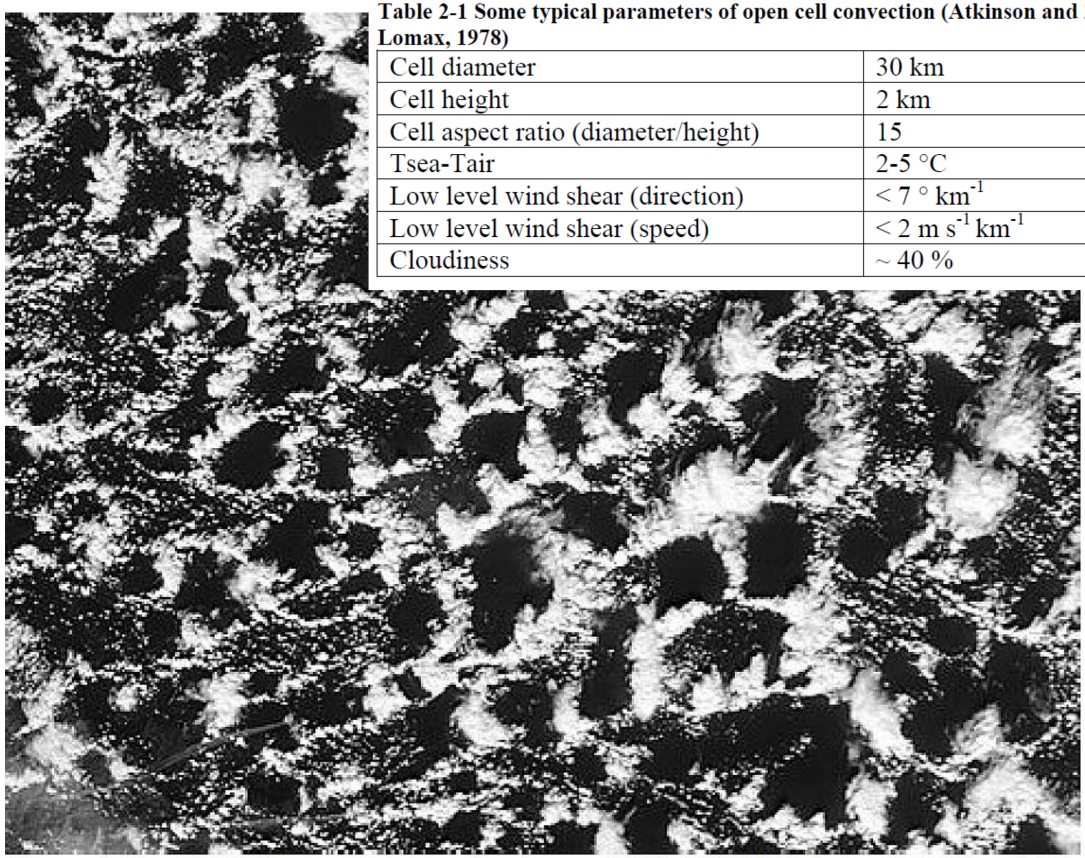


Figure 1 Open Cell Convection near the Azores (NASA MODIS Aqua Satellite – 24 January 2006 15:35 UTC - resolution 1 km; Image courtesy of MODIS Rapid Response Project at NASA/GSFC, [4])

Table 2-2 Some typical parameters of closed cell convection (Atkinson and Zhang, 1996; Agee and Lomax, 1978)

Cell diameter	24-53 km
Cell height	1,3 - 2 km
Cell aspect ratio (diameter/height)	3-28
Tsea-Tair	1.7-5 °C
Low level wind shear (direction)	< 7 ° km^{-1}
Low level wind shear (speed)	< 2 m s^{-1} km^{-1}
Cloudiness	~ 90 %

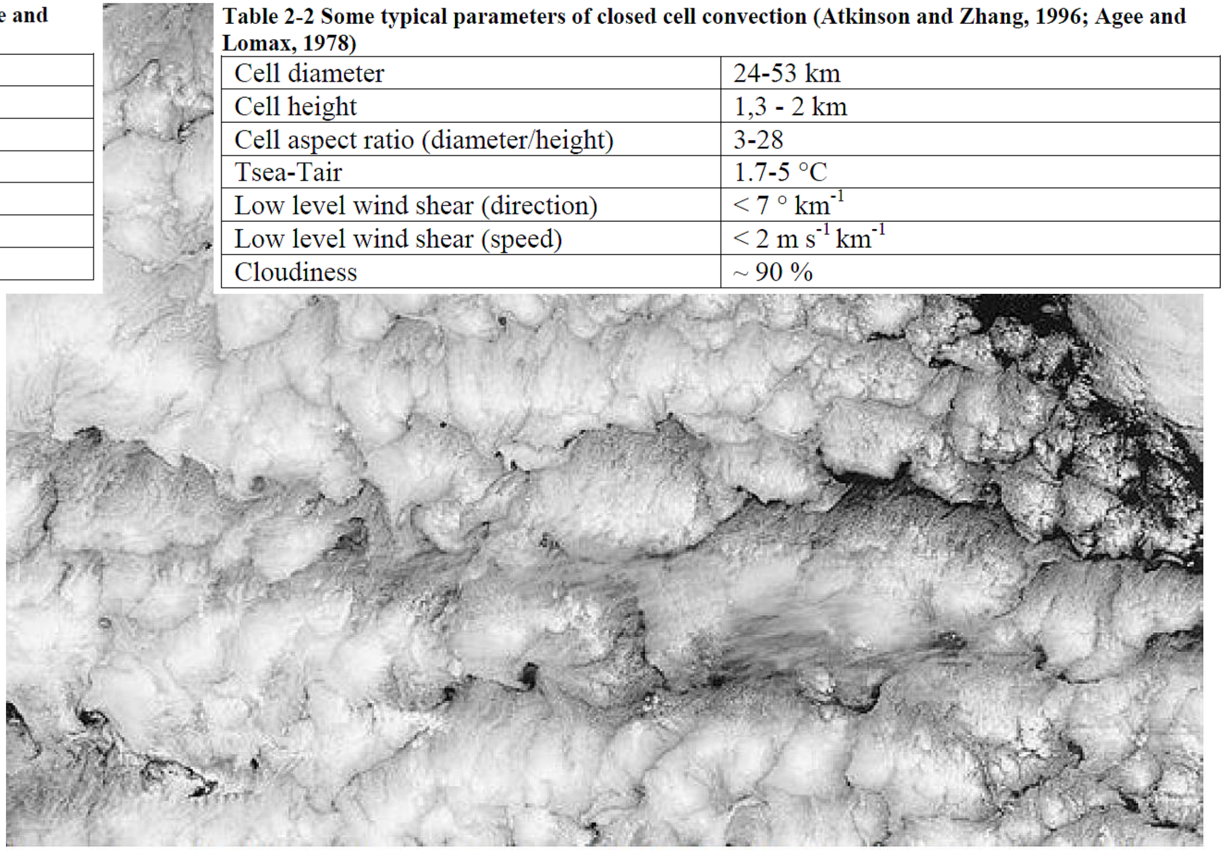
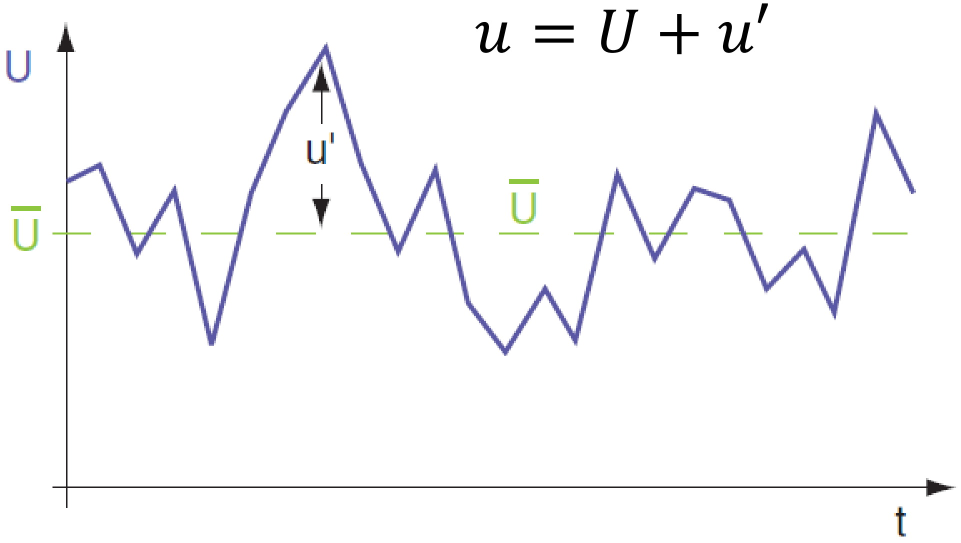


Figure 2 Closed Cell Convection near the Azores (NASA MODIS Terra Satellite – 12 April 2006 12:30 UTC - resolution 1 km; Image courtesy of MODIS Rapid Response Project at NASA/GSFC, [4])

Reynolds Decomposition: Rules



$$u = U + u'$$

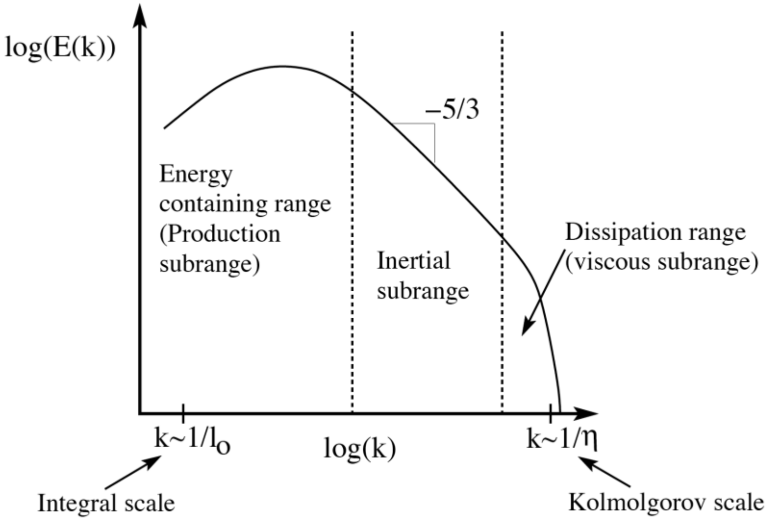
$$\overline{uu} = \overline{UU} + \overline{u'u'} + 2\overline{u'U}$$

$$\overline{UU} = (\overline{U})^2 = U^2$$
$$\overline{u'U} = \overline{u'}U = 0$$
$$\overline{u'} = 0$$

$$\underbrace{\overline{uu}}_{\substack{a\ full \\ motion}} = \underbrace{U^2}_{\substack{a\ large\ scale \\ part\ of\ the\ motion}} + \underbrace{\overline{u'u'}}_{\substack{a\ small\ scale \\ part\ of\ the\ motion}}$$

Large-eddy simulations

© by Rob Stoll

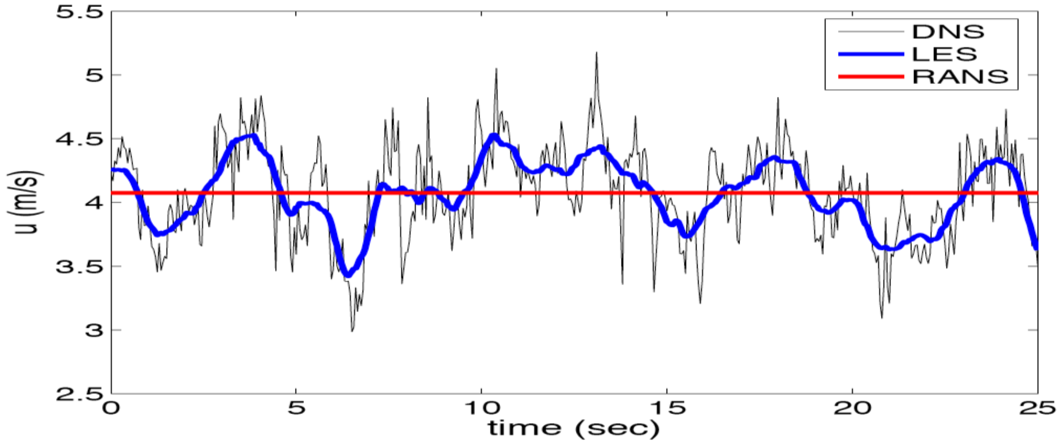


$$E(k) = c_k \epsilon^{2/3} k^{-5/3}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \quad \text{Conservation of Mass}$$

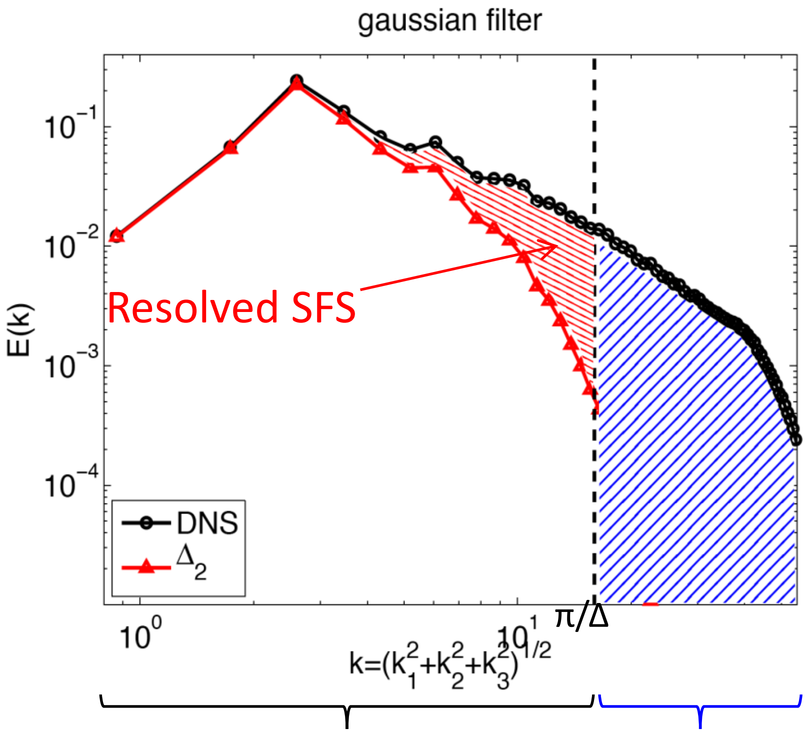
$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + F_i \quad \text{Conservation of Momentum}$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial u_i \theta}{\partial x_j} = \nu_\theta \frac{\partial^2 \theta}{\partial x_j^2} + Q \quad \text{Conservation of scalar (temp, species, etc.)}$$



Turbulence subgrid-scale closure and filtering

© by Rob Stoll



$$\phi = \widetilde{\phi} + \phi'$$

$$\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

$$\widetilde{u_i u_j} = \widetilde{(\tilde{u}_i + u_i')(\tilde{u}_j + u_j')}$$
$$= \widetilde{\tilde{u}_i \tilde{u}_j} + \widetilde{\tilde{u}_i u_j'} + \widetilde{\tilde{u}_j u_i'} + \widetilde{u_i' u_j'}$$

$$\widetilde{\tilde{u}_i \tilde{u}_j} = \underbrace{\left(\widetilde{\tilde{u}_i \tilde{u}_j} - \tilde{u}_i \tilde{u}_j\right)}_{L_{ij}} + \tilde{u}_i \tilde{u}_j$$ **-> known**

L_{ij} - Leonard term (stress)

$$C_{ij} = \widetilde{\tilde{u}_i u_j'} + \widetilde{\tilde{u}_j u_i'} \;\Rightarrow\; \text{interaction between resolved and SFSs}$$

$$R_{ij} = \widetilde{u_i' u_j'} \;\Rightarrow\; \text{SFS “Reynold’s” stress}$$ **-> unknown**

Required closure for the turbulent stress term

$$\tau_{ij} = L_{ij} + C_{ij} + R_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\nu_T \tilde{S}_{ij}$$ **-> parametrization (J. Smagorinsky)**

Nature of turbulent fluxes

$$\frac{\partial u_i}{\partial t} = -\frac{\partial u_i u_j}{\partial x_j} - \frac{1}{\rho}\frac{\partial p^*}{\partial x_i} - \frac{\partial}{\partial x_j} k \frac{\partial u_i}{\partial x_j} + \cdots$$

Decompose on resolved and unresolved parts (Reynolds approach or Keller-Friedman series: $\overline{u'} = 0, \overline{\bar{u}u'} = 0$)

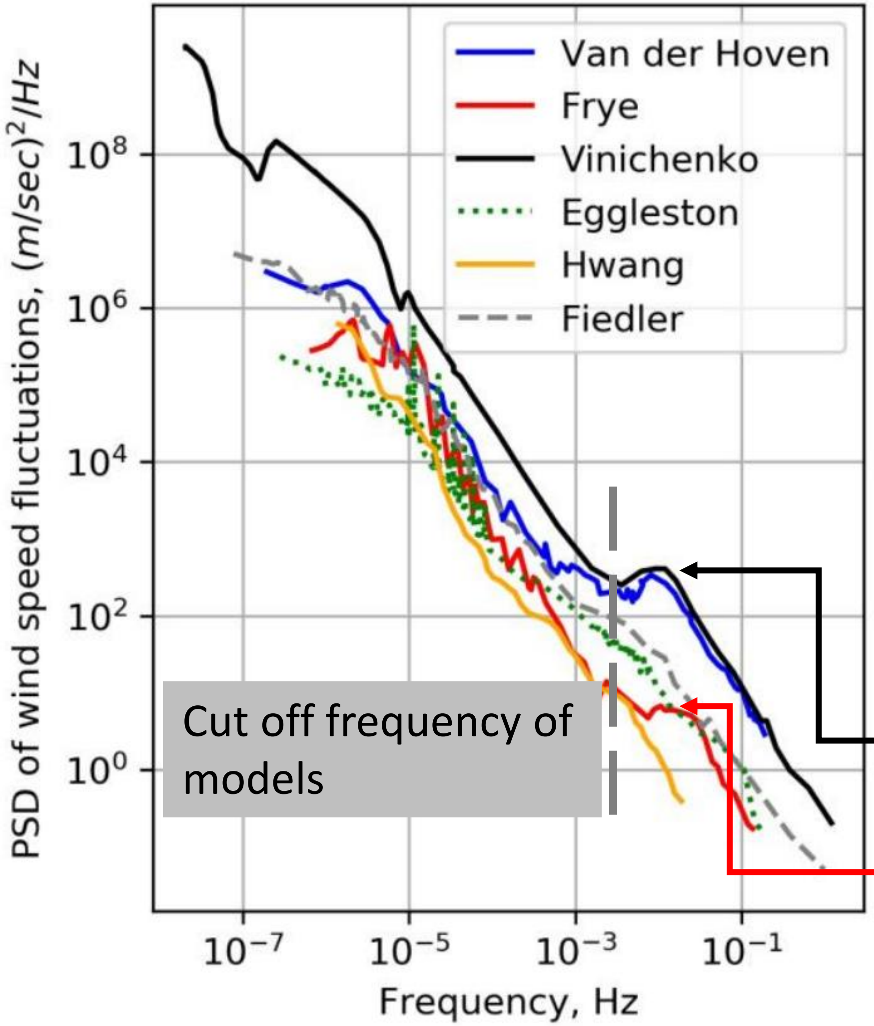
$$\frac{\partial \bar{u}_i}{\partial t} = -\left(\frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} + \frac{\partial \overline{u'_i u'_j}}{\partial x_j} + \frac{1}{\rho}\frac{\partial \overline{p^*}}{\partial x_i} + \frac{\partial}{\partial x_j} k \frac{\partial \bar{u}_i}{\partial x_j}\right)$$

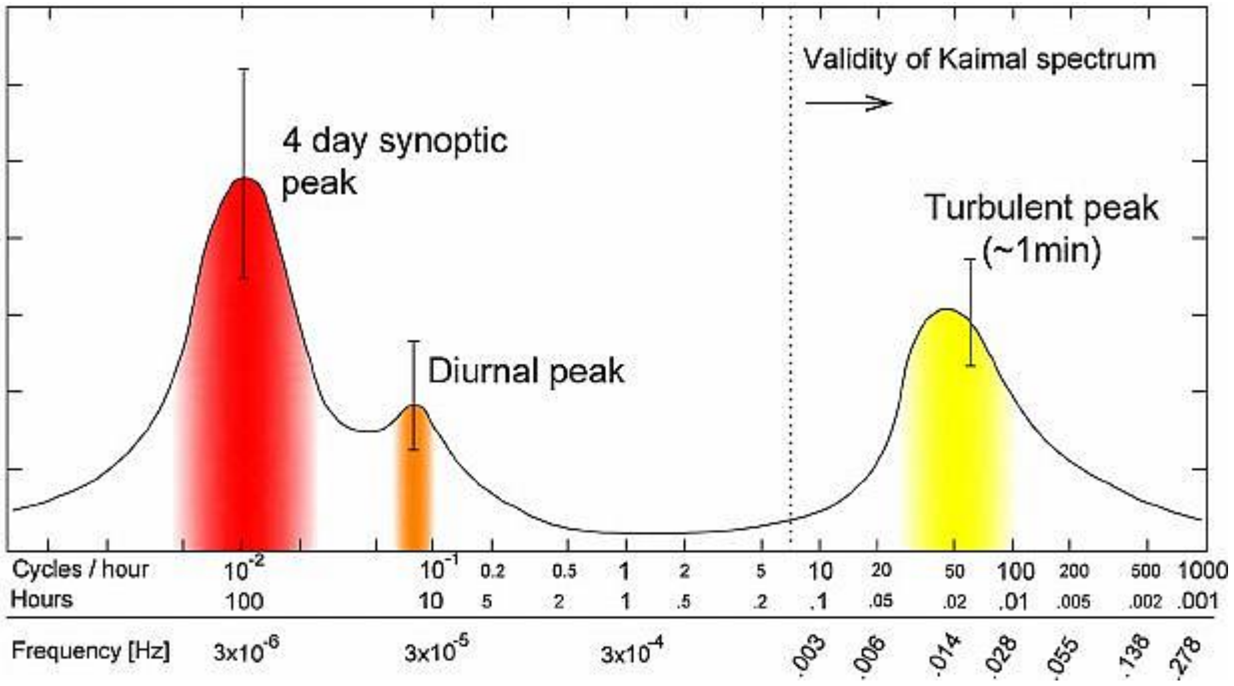
This term is supposed to represent the effect of **small-scale (unresolved)** motions on **large-scale (resolved)** motions

Flux-gradient hypothesis (**the first order closure**): Advection term is similar to diffusion term for small-scale chaotic motions (turbulence)

$$\overline{u'_i u'_j} = k_M \frac{\partial \bar{u}_i}{\partial x_j}$$

Why do models need a turbulence scheme?





Turbulent peak of energy in the PBL

Van der Hoven (1957)

No turbulent energy in the free atmosphere

doi:10.1088/1755-1315/211/1/012023

Flux-Gradient Assumption (K-theory)

Flux-gradient assumption $\underbrace{\tau_{xz}}_{turbulent\ flux} = \frac{\partial}{\partial z}\left(\overline{u'w'}\right) = 2 \underbrace{K_m}_{eddy\ viscosity} \underbrace{S_{xz}}_{velocity\ gradient\ (shear)}$

Eddy viscosity $K_m = (c_S l)^2 |S|$

Flux-gradient assumption $\tau_{\theta z} = K_\theta \frac{\partial \theta}{\partial z}$

Eddy diffusivity $K_\theta = K_m \underbrace{\mathrm{Pr}^{-1}}_{Prandtl\ number}$

$$S_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \rightarrow \frac{1}{2}\frac{\partial U}{\partial z}$$

The first order closure

The first order closure applied at large-scales:

- $\overline{u'_1 u'_2} = \overline{u'v'} = 0$
- k_M is scalar

$$\overline{u'_1 u'_3} = \overline{u'w'} = k_M \frac{\partial \overline{u_1}}{\partial x_3} = k_M \frac{\partial \bar{u}}{\partial z}$$

$$\overline{u'_2 u'_3} = \overline{v'w'} = k_M \frac{\partial \overline{u_2}}{\partial x_3} = k_M \frac{\partial \bar{v}}{\partial z}$$

FOC: 2 moments + 1 TKE equation (optional) + 1 TKE dissipation equation (optional)

Mellor G. L., & Yamada T. (1982). Development of a turbulence closure model for geophysical fluid problems. *Reviews of Geophysics*, *20*(4), 851–875. https://doi.org/10.1029/RG020i004p00851

A side note: Higher-order closures (play with unknowns)

SOC: 6 + 2 pressure + 1 TKE + 1 dissipation

$$m_{ij}^{(2)} = \overline{u'_i u'_j}$$

$$\frac{\partial m_{ij}^{(2)}}{\partial t} = -\left(\frac{\partial \bar{u}_i m_{ij}^{(2)}}{\partial x_j} + \frac{\partial \overline{\left(u'_i m_{jk}^{(2)}\right)}}{\partial x_j}\right) + P^{(2)} + \epsilon$$

TOC: 30 + 9 pressure + 1 TKE + 1 dissipation

$$m_{ijk}^{(3)} = \overline{\left(u'_i m_{jk}^{(2)}\right)}$$

$$\frac{\partial m_{ijk}^{(3)}}{\partial t} = -\left(\frac{\partial \bar{u}_i m_{ijk}^{(3)}}{\partial x_j} + \frac{\partial \overline{\left(u'_i m_{jkl}^{(3)}\right)}}{\partial x_j}\right) + P^{(3)} + \epsilon$$

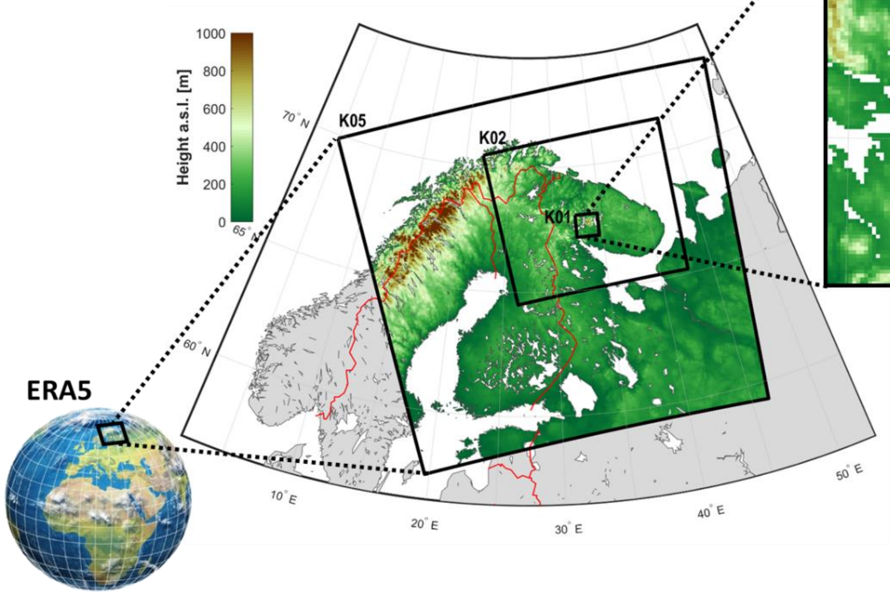
The surface layer

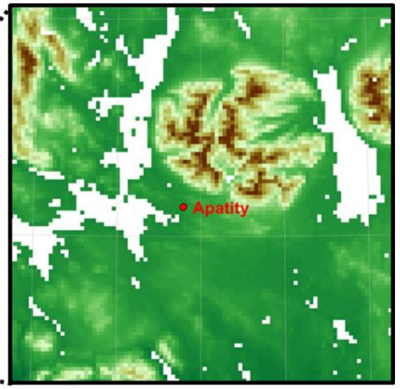
Let $u_* = \left(\overline{u'w'} + \overline{v'w'}\right)^{1/2}$ be the surface friction velocity, or the momentum flux at the surface

$k_M \left|\frac{d\bar{u}}{dz}\right| = u_*^2$ Law of the wall

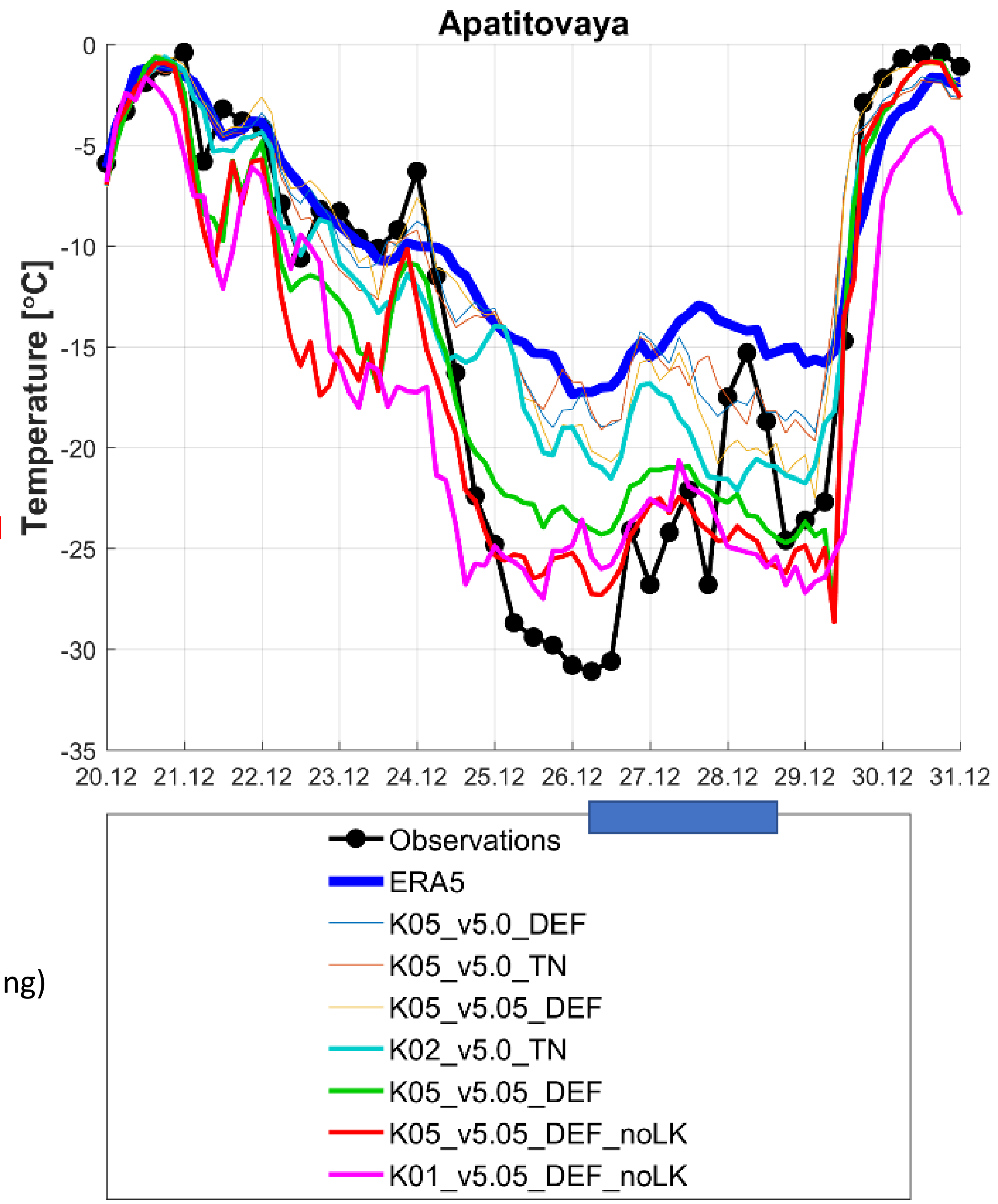
$k_M \propto u_* L = kzu_*$ In the Prandtl layer, just by a dimensional analysis (no physics); $k \approx 0.4$ – the von Karman constant, z – the height above surface

$\phi_M = \left|\frac{d\bar{u}}{dz}\right| \frac{kz}{u_*} = 1$ In the non-stratified flow





Only the simulations with new boundary layer physics, high spatial resolution and removed lakes **adequately reproduced the observed cold spell** (K02/K01_v5.05_DEF_noLK run)



Downscaling model chain

- Limited-area mesoscale model COSMO-CLM
- Dynamic downscaling of ERA5 reanalysis data (available at grid with 30 km grid spacing)
- Chain of nested domains with grid step 5, 2 and 1 km (K05, K02 and K01)
- Simulation period 20-30 December 2017
- Computations at Lomonosov-2 supercomputer of Lomonosov Moscow State University

Turbulence Scheme in MUSC

$$\frac{\partial \theta_V}{\partial t} = \frac{\partial}{\partial z} K_H \frac{\partial \theta_V}{\partial z} + F_T$$

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial z} K_M \frac{\partial U}{\partial z} + F_U$$

Comments: F_U is a given external forcing; F_T is the external cooling or warming rates.

If the turbulence scheme causes difficulties, one shall look at eddy diffusivity coefficients
... and further at the mixing length scale

$$l_{M,H}$$

The parametrized form of the TKE equation is

$$\frac{\partial E_k}{\partial t} = K_M S^2 - K_H N^2 + 2K_M \frac{\partial E_k}{\partial z} - \epsilon$$

$$\epsilon = c_d \frac{(E_k)^{3/2}}{l_M}$$

$$K_M = K_H = l_{M,H} \sqrt{E_k}$$

The mixing length scales $l_{M,H}$ absorbed all coefficients.

Energy-Flux – Balance scheme

$$\frac{DE_K}{Dt} = \frac{\partial}{\partial z} K_E \frac{\partial E_K}{\partial z} + \tau_x \frac{\partial U}{\partial z} + \tau_y \frac{\partial V}{\partial z} + \beta \tau_\theta - \frac{E_K}{t_T}$$

$$\frac{DE_P}{Dt} = \frac{\partial}{\partial z} K_E \frac{\partial E_P}{\partial z} - \beta \tau_\theta - \frac{E_P}{C_P t_T} \qquad E_P = \left(\frac{\beta}{N}\right)^2 E_\theta = \frac{1}{2}\left(\frac{\beta}{N}\right)^2 \overline{\theta'^2} = c_{Ep\theta} N^{-2} \overline{\theta'^2}$$

The relaxation (prognostic) equation for the dissipation time scale

$$\frac{dt_T}{dt} = t_T(t) - t_T(t-\delta t) = \min\left(0.2, \frac{\delta t}{t_{TE}}\right)(t_{TE}(t) - t_T(t-\delta t))$$

t_{TE} is an equilibrium time scale, associated with the length scale $l = t_{TE}\sqrt{E_K}$, its diagnostic expression is given by

$$t_{TE} = kz \frac{1}{\sqrt{E_K} + C_\Omega \Omega z} \left(\frac{E_K}{\tau}\right)^{\frac{3}{2}} \left(1 - \frac{\Pi}{\Pi_{inf}}\right)$$

the Earth rotation set C_Ω= 0.

Energy-Flux – Balance scheme

The turbulence diffusion coefficients are given as

$$K_M = 2C_\tau A_z E_K t_T$$

$$K_H = 2C_F \left(1 - C_\theta \frac{E_P}{A_z E_K}\right) A_z E_K t_T$$

In the MUSC realization, the anisotropy was kept constant, $A_z = 0.2$.

A measure (energy-based) of the static stability is given by $\Pi = \frac{E_P}{E_k} = \frac{C_P R_f}{1 - R_f}$

where $\Pi_{inf} = 0.14$ is its asymptotic limit.

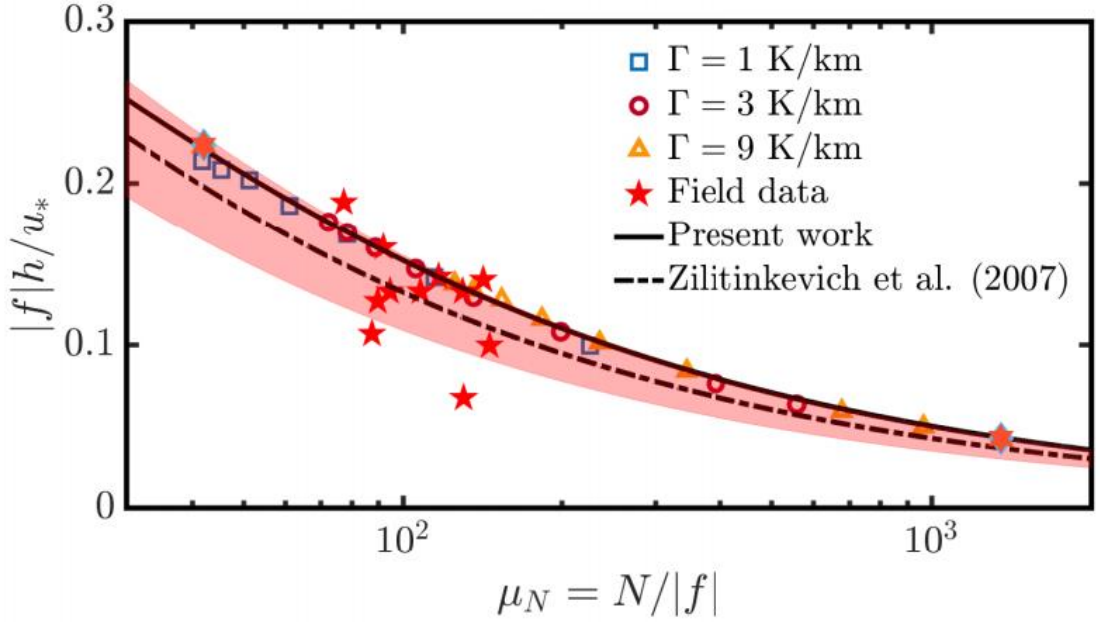
The Ekman layer – a link to the ABL depth

The plain boundary layer is unbounded, $h \propto k u_* x^{1/2}$

The environmental (Ekman) boundary layer is bounded, $h \propto \sqrt{\frac{k_M}{f}} = \sqrt{\frac{k h u_*}{f}} = c_R \frac{u_*}{f}$, $c_R \approx 0.65$

The Ekman PBL depth is directly proportional to the surface turbulent flux, $h \propto u_*$

$C_R =$



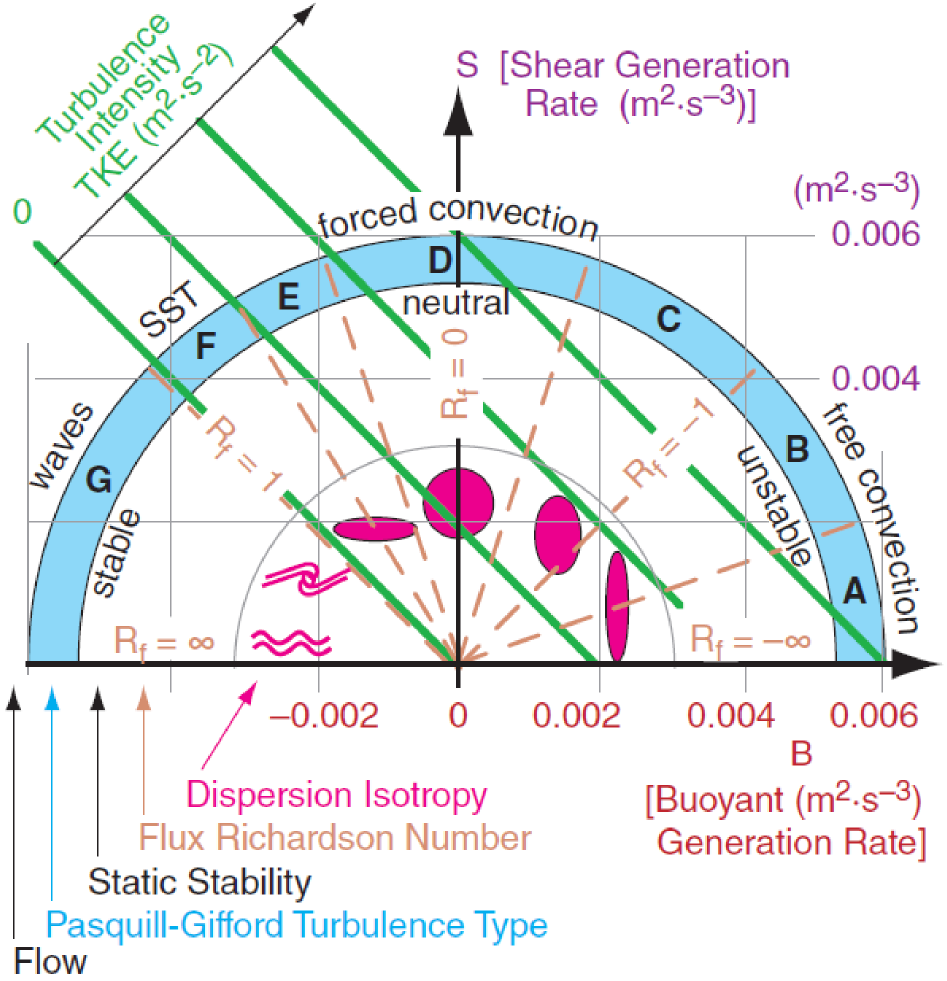
Stratification of free atmosphere

Conventionally neutral PBL. The fact that free atmosphere impacts the PBL depth tells that surface flux is not strictly related to the depth of the mixed layer and **can be enhanced without making the PBL deep**.

Liu L., Gadde S. N., & Stevens R. J. A. M. (2020). Geostrophic drag law for conventionally neutral atmospheric boundary layers revisited. *Quarterly Journal of the Royal Meteorological Society*, qj.3949. https://doi.org/10.1002/qj.3949

Zilitinkevich S., Esau I., & Baklanov A. (2007). Further comments on the equilibrium height of neutral and stable planetary boundary layers. *Quarterly Journal of the Royal Meteorological Society*, *133*(622). https://doi.org/10.1002/qj.27

Stratification and stability



Turbulent kinetic energy

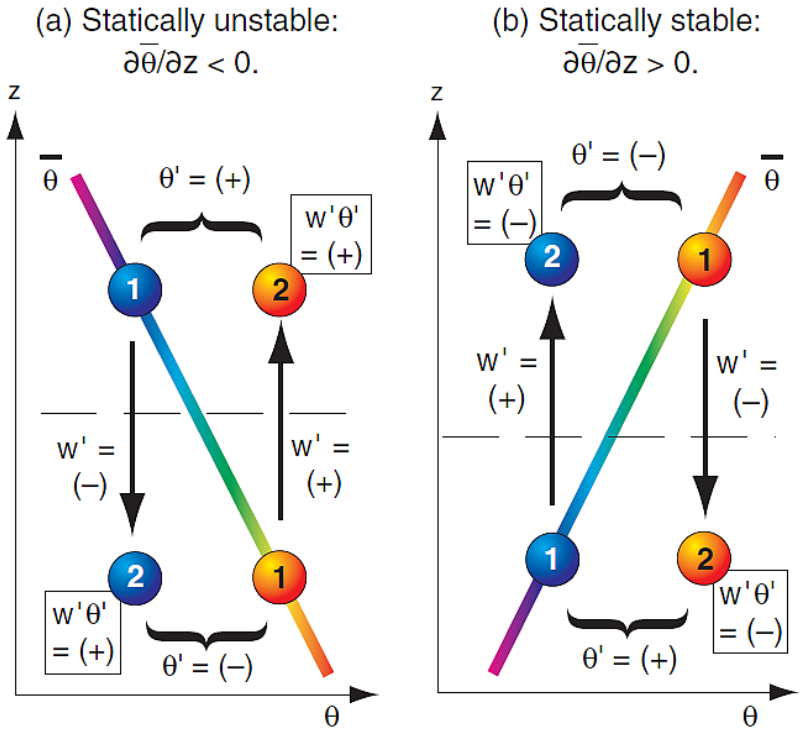
$$E_K = \frac{1}{2}(\boldsymbol{u} \cdot \boldsymbol{u}^T)$$

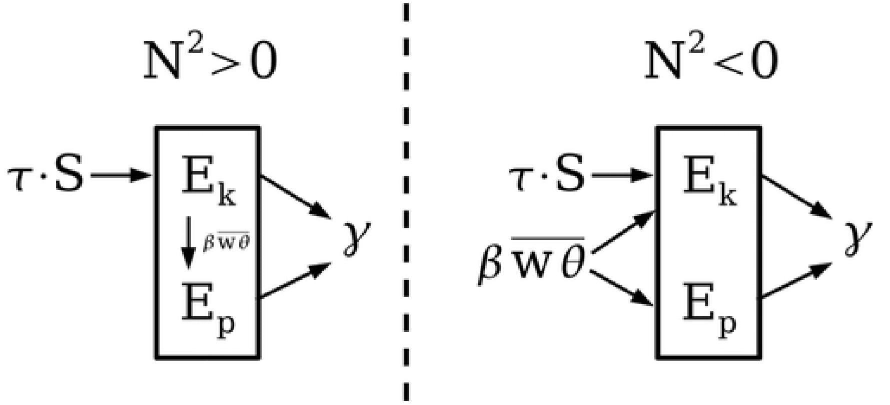
Turbulent potential energy

$$E_\theta = \frac{1}{2}(\theta'\theta')\frac{g}{\theta_0}\left(\frac{\partial\theta}{\partial z}\right)^{-1}$$

Turbulent dissipation

$$\gamma = C_\gamma \frac{E^{3/2}}{l}$$

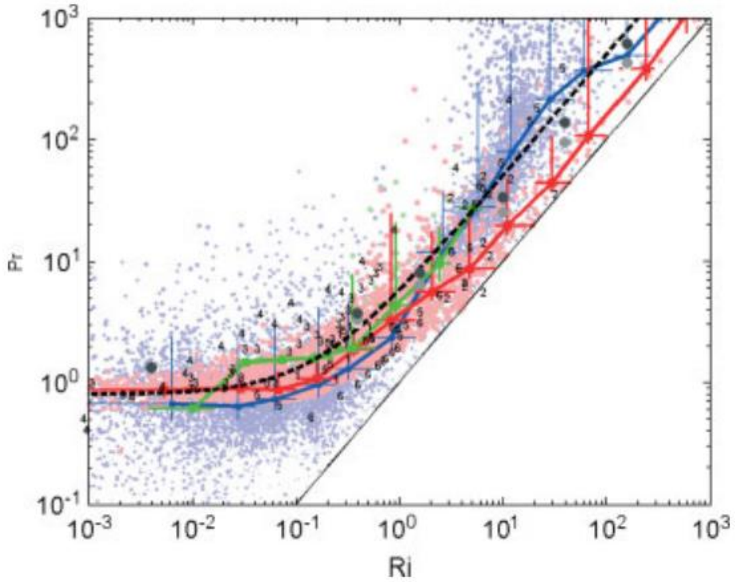




Temperature equation
(approaching to the temperature bias)

$$\frac{\partial T}{\partial t} = -\left(\bar{u_i}\frac{\partial T}{\partial x_i} + \frac{\partial \overline{u_i'T'}}{\partial x_i} + \frac{\partial}{\partial x_i}k_T\frac{\partial T}{\partial x_i}\right) + \frac{1}{\rho c_p h}\sum div(Q_k)$$

- Assume PBL conditions, $\frac{\partial \overline{u_i'T'}}{\partial x_i} = \frac{\partial \overline{w'T'}}{\partial z}$
- Assume no lateral fluxes, $\bar{u_i}\frac{\partial T}{\partial x_i} = 0$
- Apply to a bulk surface layer of the depth h



1. Monin-Obukhov similarity theory (MOST)
   - Equilibrium conditions, $\frac{dT}{dt} = 0$
   - Arrive to flux-gradient relationship

$$k_T\left|\frac{d\bar{T}}{dz}\right| = \mathrm{T}_* = \overline{w'T'}$$

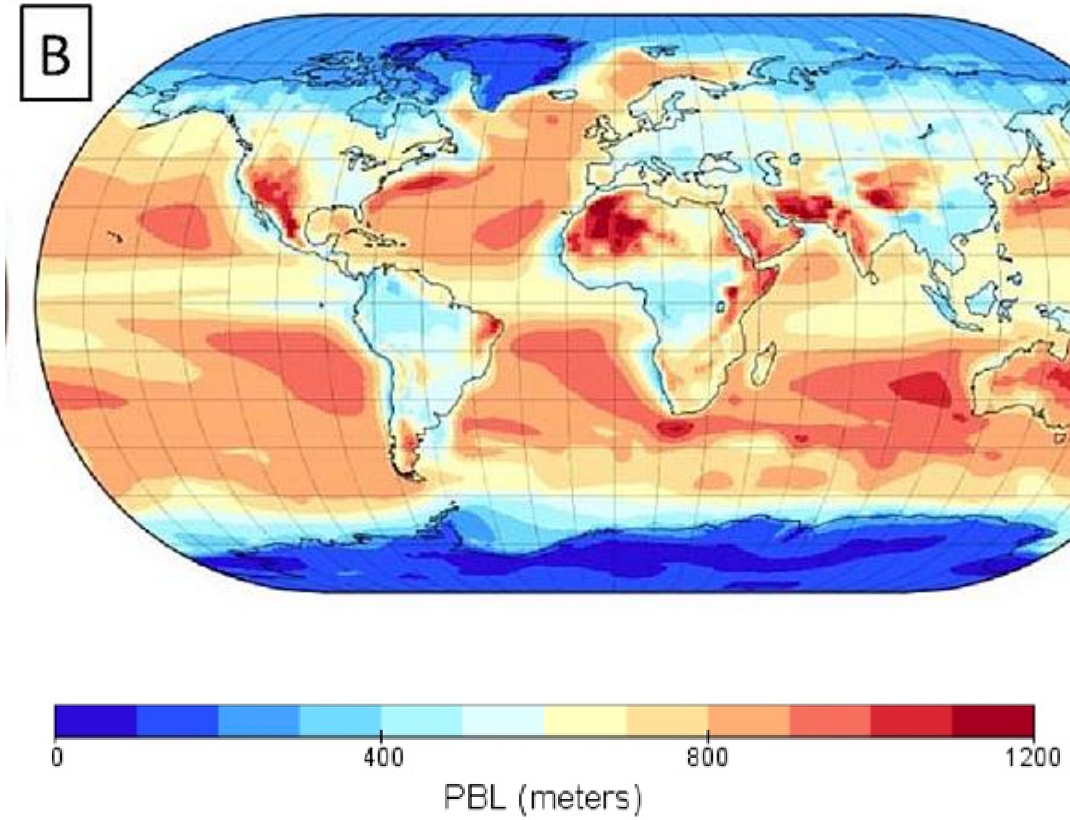
Let us define the Prandtl number (unknown!)

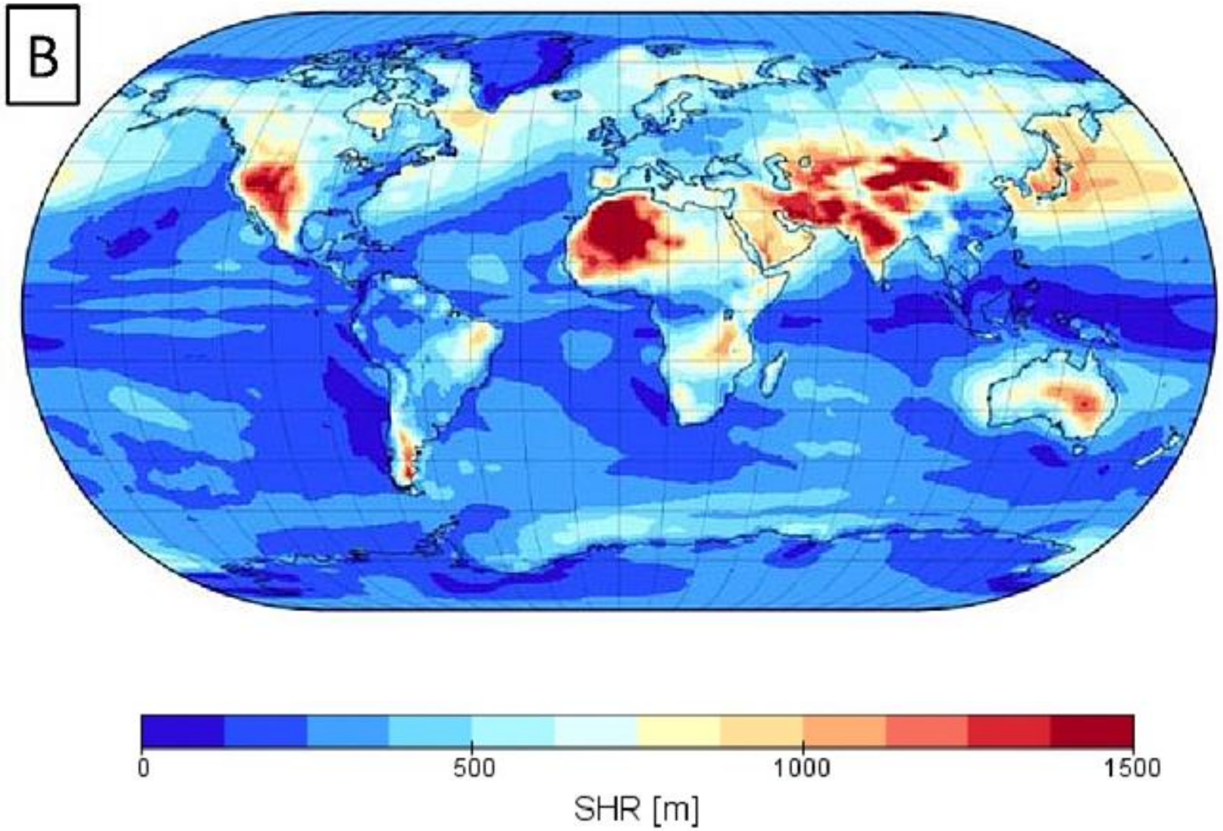
$$Pr = \frac{k_M}{k_T} = \frac{\left|\frac{d\bar{T}}{dz}\right| u_*^2}{\left|\frac{d\bar{u}}{dz}\right| \mathrm{T}_*}$$

$$k_T = \frac{k_T}{k_M}k_M = \frac{1}{Pr}k_M = \frac{1}{Pr}\frac{u_*^2}{\left|\frac{d\bar{u}}{dz}\right|}$$

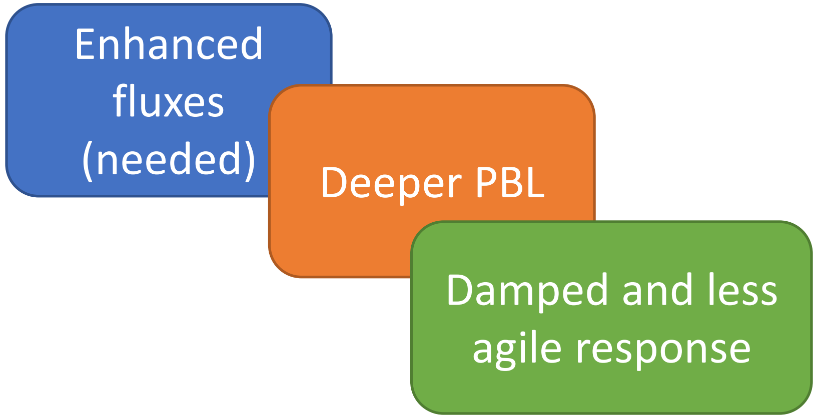
Focus of almost all studies is on phenomenology of $\mathbf{Pr}(\boldsymbol{Ri})$

Climatology of the PBL





Variable ABL: Climate effects



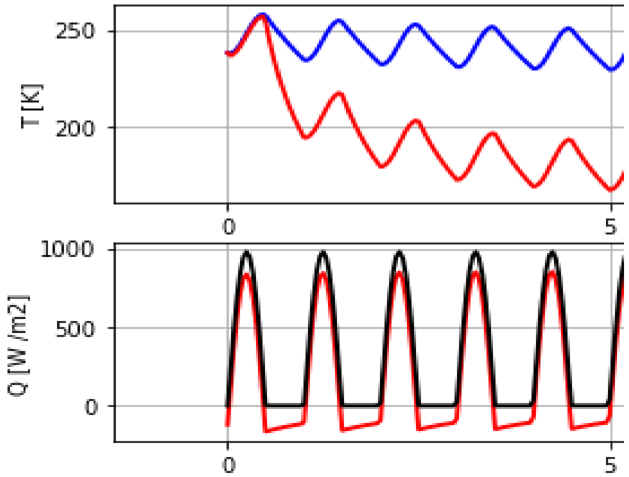
$$C\frac{dT}{dt} = Q = \underbrace{aS_0}_{\substack{solar\ flux \\ \max(0, sin\omega t)}} - \underbrace{(A+BT)}_{\substack{thermal\ flux \\ linearization}} + \underbrace{F}_{\substack{climate\ forcing \\ \propto t}} + \underbrace{N(0,\sigma_T)}_{\substack{additive\ noise \\ (weather)}}$$

$$C = \rho c_p h = \begin{cases} h_{SBL} \sim 100\ m, Q < 0 \\ h_{CBL} \sim 10 h_{SBL} \sim 1000\ m, Q \geq 0 \end{cases}$$

Different cases:

1. "Fast" system, $\tau < \tau_Q$, i.e., the system adopts to the forcing
2. "Slow" system , $\tau > \tau_Q$, i.e., the system integrates the forcing
3. Alternating system, incomplete adaptation and new equilibrium

1. PBL scheme could be asymptotically (in a long relaxation run) correct – **all bias is accumulated in a few hours**
2. Variable forcing correlated with variable PBL leads to **different model climatology**



$h = const$

$h = H(t)$

ABL as a factor of Climate change